Using Weibull Distribution Analysis to Evaluate ALARA Performance

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Abstract - As Low as Reasonably Achievable (ALARA) is the underlying principle for protecting nuclear workers from potential health outcomes related to occupational radiation exposure. Radiation protection performance is currently evaluated by measures such as collective dose and average measurable dose, which do not indicate ALARA performance. The purpose of this work is to show how statistical modeling of individual doses using the Weibull distribution can provide objective supplemental performance indicators for comparing ALARA implementation among sites and for insights into ALARA practices within a site. Maximum likelihood methods were employed to estimate the Weibull shape and scale parameters used for performance indicators. The shape parameter reflects the effectiveness of maximizing the number of workers receiving lower doses and is represented as the slope of the fitted line on a Weibull probability plot. Additional performance indicators derived from the model parameters include the 99th percentile and the exceedance fraction. When grouping sites by collective total effective dose equivalent (TEDE) and ranking by 99th percentile with confidence intervals, differences in performance among sites can be readily identified. Applying this methodology will enable more efficient and complete evaluation of the effectiveness of ALARA implementation.

Key words: ALARA, Weibull distribution, statistical modeling, percentile, exceedance fraction, occupational radiation exposure, radiation protection

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1. INTRODUCTION

ALARA is a radiation safety principle for minimizing risk of exposure to radiation by utilizing all practical, cost effective methods. ALARA is also a regulatory requirement for exposed worker population in the United States. It is important to note that the ALARA principal includes the reduction of the collective exposure as low as can be reasonably achieved, as well as keeping the dose to individual workers as low as reasonably achievable. The indicators most commonly used in radiation protection such as collective dose and average measurable dose do not consider the distribution of dose among the individuals receiving dose, and therefore do not support both aspects of the ALARA principal.

The Department of Energy (DOE) and the U.S. Nuclear Regulatory Commission (NRC) are committed to protecting the exposed worker population from any potential health outcomes related to occupational exposure by keeping their doses as far below the permissible exposure limits as possible. Statistical modeling with the Weibull distribution can provide quantitative measures of the distribution of doses among workers at a site that can be used to track ALARA effectiveness over time and/or to compare differences among sites.

1.1. Goals of Analysis

The first goal of the statistical analysis of the TEDE dose distributions was to illustrate the utility of the Weibull distribution for describing the effectiveness of applying the ALARA principle to individuals exposed to radiation at a site. The parameters of the Weibull distribution were used to summary statistics and graphical representations that reflect the extent to which a site is effective in applying ALARA.

1.2. Minimum Inclusion Threshold

The ALARA principal is applied to individuals in the workplace by radiation protection personnel in an occupational setting. The radiation protection program focuses on using basic principles such as time, distance and shielding to reduction exposure, as well as optimizing the efficiency of workers and advanced planning of the work to be performed. Not all individuals monitored at a facility are actively engaged in work involving radiation or radioactive materials. Many are monitored for security or legal reasons and may not be exposed, or receive small doses due to their office location at the worksite. Individuals that fall into this category are not actively engaged in radiation work that falls directly under the purview and control of the radiation protection and ALARA staff.

It is therefore desirable to limit analysis to those individuals with a dose that exceeds a "minimum inclusion threshold" (MIT). Having an MIT is conceptually equivalent to saying that only those individuals with a dose above the MIT at the end of the year are considered exposed worker population and are, therefore, subject to the application of the ALARA criteria on an individual basis. Since it not possible to know at the beginning of a year which workers will have doses above the MIT at the end of a year, the exclusion is done retrospectively. An annual dose of 0.1 mSv was chosen as the MIT for a radiation worker; details concerning the selection of this value appear in Appendix A. Future changes in monitoring technology or administrative practices and procedures might indicate the need to reexamine this value. However, 0.1 mSv is a fixed value for all analyses described in this report.

2. MATERIALS AND METHODS

2.1. Weibull Distribution

The two-parameter Weibull distribution (Weibull, 1939) has been widely used to describe positive data due to its versatility from various combinations of its shape and scale parameter. Weibull (1951) described the application to the analysis of strength of materials. Murthy et al. (2004) list a large number of examples from a variety of applications that use the Weibull probability mode. These examples are as diverse as rain drop size, tensile strength of optical fibers, traffic conflict in expressway merging, thermo-luminescence glow, and flight load variation in helicopters. Use of the Weibull model is especially prevalent in reliability analysis for manufacturing. The Weibull distribution was selected on empirical grounds for the application of dose distributions of workers at nuclear sites.

The Weibull probability density function (Johnson et al., 2004) with shape parameter α and scale parameter β is as follows:

$$f(x) = \alpha/\beta\ [x/\beta]^{\alpha-1} \exp[-(x/\beta)^{\alpha}] \qquad (1)$$

The corresponding cumulative distribution function is this:

$$P[X \leq x] = F(x) = 1 - \exp(-[x/\beta]^{\alpha}),\ \alpha > 0,\ \beta > 0. \qquad (2)$$

The Exceedance Function $S(x) = P[X > x] = 1 - F(x)$; it is also known as the Survival or Reliability Function. In this application $S(x)$ is the proportion of doses that are greater than x. When the shape parameter is less than one, the probability density function is asymptotic to the y-axis.

The shape parameter α is equal to the slope of the regression line in a Weibull probability plot and is unitless. For the application of site radiation dose distributions discussed in this report, the Weibull model appears to fit the distributions effectively when the shape parameter is less than one. When $\alpha = 1$ the Weibull distribution simplifies to the negative exponential distribution. Shape parameters below 2.6 indicate positive skewness (right tail), and above 3.7 negative skewness (left tail) occurs. Changes in β have the same effect as changing the scale on the y-axis. If the y-axis scale remains constant and β is increased, the distribution is stretched to the right and the height decreases. Similarly, a decrease in β pushes the distribution towards the left and increases the height. Figures 1 and 2 demonstrate how changes in the shape of the Weibull curve are related to the shape and scale parameters.

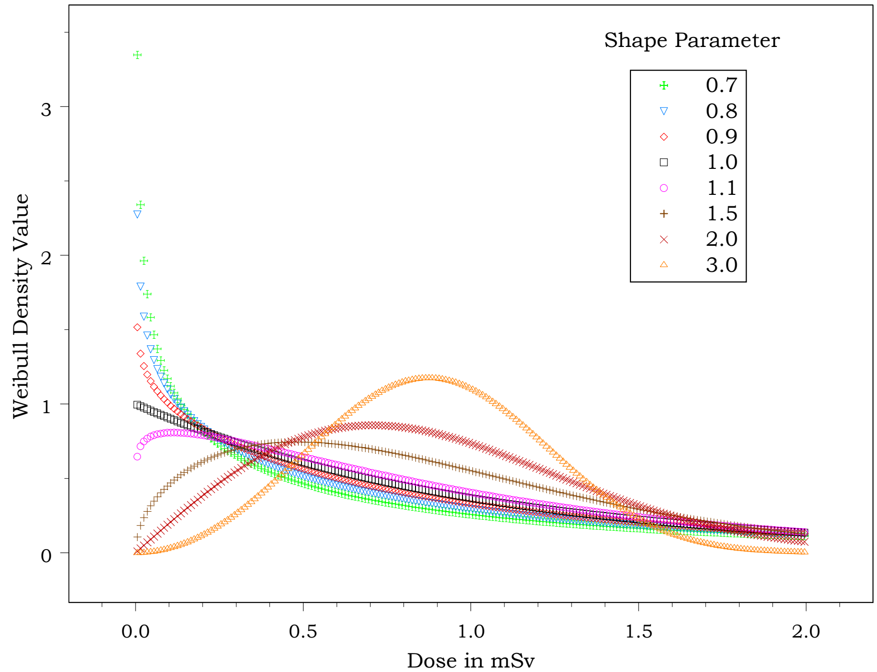


Figure 1: Weibull Density for Scale Parameter of 1.0 and Shape Parameters As Shown

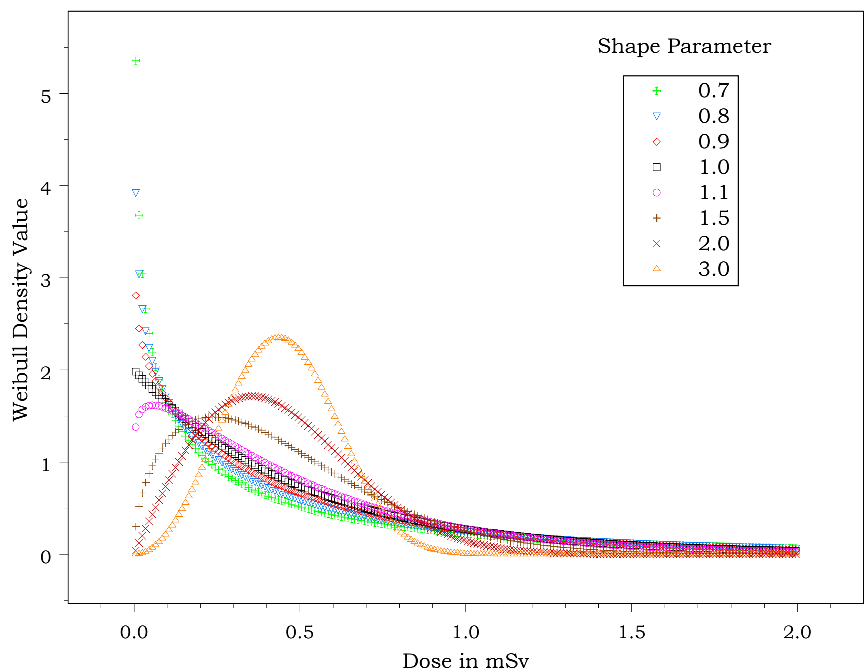


Figure 2: Weibull Density for Scale Parameter of 0.5 and Shape Parameters As Shown

All analyses in this study modeled distributions of workers at a site with annual dose above the MIT of 0.1 mSv, and x was defined as follows: x = (TEDE – MIT). Maximum likelihood (ML) methods were used to estimate the shape and scale parameters values and their standard errors based on individual doses from a specific site and time period, generally a year.

2.2. Indicators of ALARA Effectiveness

Statistical modeling of worker dose distributions provides parameters that can be used to derive indicators of ALARA performance. In addition to the Weibull shape parameter, further indicators, which can be customized to a site, include an upper percentile or an exceedance fraction. Customized probability plots provide visual evidence of ALARA effectiveness and can include a tailored reference line that allows comparison among similar sites or over time for a given site.

2.2.1. ALARA Interpretation of Shape Parameter

A situation that would seem to characterize successful ALARA application is the case where the shape parameter $\alpha < 1$, which means the mode of the distribution is at the MIT and the density function is a decreasing function of x. As an ALARA performance indicator, the value of α should be examined, and the site should be considered as not exhibiting effective ALARA implementation if this value is greater than one. The slope of the Weibull regression line on the customized probability plot is the negative of the value of α.

2.2.2. An Upper Percentile and Tolerance Limit

Let x_p denote the $100p^{th}$ percentile of a dose distribution, i.e. 100p% of the doses are less than or equal to x_p. For the Weibull distribution x_p is the value of x such that $F(x_i, \alpha, \beta) = 1 - \exp[-(x/\beta)^{\alpha}]=p$, which leads to $\log(x_p) = \log(\beta) + (1/\alpha)[-\log(1-p)]$. For a given significance level γ (e.g., γ =0.95), an upper confidence limit for x_p, $UX(p, \gamma)$ is the value of x such that one is $100\gamma\%$ confident that 100p% of the doses are below this value. Correspondingly, 100(1-p)% of the doses exceed $UX(p, \gamma)$. $UX(p, \gamma)$ is also referred to as an upper tolerance limit. The $100p^{th}$ percentile is used to characterize the upper tail of the dose distribution, and the upper tolerance limit describes the uncertainty in the point estimate for x_p for a given dose distribution.

For the current application p=0.99 and γ=0.95. Fitted x_p (99^{th} percentiles) of the dose distributions and their 100γ (95%) confidence limits were calculated as linear combinations of ML parameter estimates and standard errors. Details of obtaining ML estimates and using them to calculate the percentiles and their confidence limits appear in Appendix B. The fitted 99^{th} percentiles are shown graphically on the Weibull probability plots using dashed horizontal lines at S(x) = 100(1–p) = 0.01 or 1%. The solid line in the plot representing the Weibull fit intersects this horizontal line at the ML estimate of the 99^{th} percentile.

The estimates of the percentiles and confidence limits provide a tool that can be used to determine whether a specific distribution is consistent with ALARA objectives and to compare distributions over time or among different sites. Sites can be ranked by their 99^{th} percentiles themselves or by the tolerance limits. If, for example, health physicists would like to see 99% of the annual doses less than a specified limit La with 95% confidence, then the value of the tolerance limit UX(0.99,0.95) should be less than La, where La might be 2.5 mSv ($1/20^{th}$ of the annual occupational exposure limit.)

2.2.3. Determining an Exceedance Fraction

The exceedance fraction describes the proportion of doses that exceed a specified limit L and can be used as an informal measure of performance for ALARA purposes. The exceedance fraction, converted to percent exceedance by multiplying by 100, is an alternative ALARA indicator to 99th percentile that can be useful when there is a particular value that is of interest for L. If a health physicist can provide a value of L and a maximum proportion q (e.g. 0.05) of doses that may exceed the limit, then the exceedance fraction and confidence limits can be used as an informal measure of compliance for ALARA purposes. For the Weibull distribution the exceedance fraction for limit L is $s = S(L) = \exp[-\theta\ (x/L)^{\alpha}]$, where $\theta = 1/\beta$. ML estimates of the exceedance fraction and its $100\gamma\%$ confidence limits are calculated and are shown in the Weibull probability plot as a vertical solid line segment at the value x=L. Details of calculating exceedance fraction and its confidence limits appear in Appendix B. The upper confidence limit for the exceedance fraction is an indicator that can be used to identify or compare ALARA effectiveness. If the upper confidence limit $Us(L, \gamma)$ is below the maximum proportion q, then we are $100\gamma\%$ confident that at most 100 $ULs(L, \gamma)\%$ of the doses exceed the limit L, and the distribution of doses is in compliance with the ALARA criteria set by the health physicist. If, however, the lower confidence limit $Ls(L, \gamma)\%$ is greater than q, then this indicates that the distribution of doses does not meet the ALARA limit.

A relationship exists between the upper percentile x_p and the exceedance fraction s. In theory, when $q = 1-p$ and L = La, if the upper CL for x_p is less than L, then the upper CL for s should be less than q. This is not an exact mathematical result since estimated confidence limits for both x_p and s are based on large sample properties of ML estimates. Because large sample properties of ML estimates are used in this application, it is not appropriate to apply these Weibull distribution methods to a dataset containing doses with fewer than 20 workers.

2.2.4. Creating a Weibull Probability Plot

A probability plot is a graphical method that can be used to examine a set of data and evaluate how well it fits a specified statistical distribution (Waller and Turnbull, 1992). The general technique is to plot the empirical quantiles from the ordered data against the theoretical quantiles determined from the specified statistical distribution to verify whether or not the points fall on a straight line, with departures from the line indicating departures from the distribution. The customized Weibull probability plots presented here show the exceedance fraction S(x) on the vertical axis.

The Weibull probability plots in Figures 3-6 were created by first ordering the distinct dose values x from smallest to largest, where an ordered dose is indicated by u. The Exceedance function $S(u) = \exp([-(u/\beta)^{\alpha}]$ was transformed into the equation of a straight line defined by $y_i = \alpha \ln(\beta) - \alpha \ln(u_i)$, where u_i is an ordered dose and $\ln(u_i)$ is the horizontal axis value. ML methods were used to obtain estimates for α and β. The steps used to create the ordered pairs ($\ln(u_i)$,$-\ln(-\ln(u_i))$) for graphing appear in Appendix B; these points are shown as open circles in the plots. The Weibull regression line appears as a solid black line with slope $-\alpha$ and intercept $\alpha \ln(\beta)$ at u=1. If the data follow the Weibull distribution, the points will be close to the straight line. For convenience, labels on the horizontal and vertical axes were adapted to match unadjusted doses and percent exceedance for values of interest, respectively.

Three lines were added to the Weibull probability plots to provide additional information of value in assessing ALARA. The first of these lines was a blue horizontal dashed line that crosses the solid Weibull line at the ML estimate of the 100pth percentile of the dose distribution. This dashed line is drawn from $S(X) = 100(1-p) = 1\%$ for our choice of p=0.99. The confidence interval for the estimated 99th percentile appears as a solid red segment on the dashed line with a cross bar at the percentile value. Also shown in the Weibull probability plots are the ML estimates of the exceedance fraction for the limit L= 2.5 mSv and its 95 % confidence limits, with the confidence interval indicated as a green vertical segment that intersects the Weibull line at x = 2.5 mSv in MIT-adjusted dose units. A horizontal line (shown in bright green with alternating dots and dashes) drawn from the point of intersection to the y-axis cuts the y-axis at the percent exceedance for x = L. The third added line is a reference line that is useful in comparing ALARA effectiveness between graphs of different time periods or sites. In our probability plots the reference line represents an ALARA goal of 99% of the doses being less than a reference value of 2.5 mSv and has a slope of one. Details of creating the reference line appear in Appendix B.

3. RESULTS

3.1. Description of Figures

The Weibull probability plots can provide a visual comparison of a site's dose distributions over time or of the distributions among sites of interest. Figures 3-9 provide examples of how the Weibull analysis approach can be used in a variety of situations comprising both large and small sites and sites with job tasks and radiation sources that vary considerably. Figure 3 introduces the probability plots and shows the location of the slope, 99th percentile, and percent exceedance with explanations. Figures 4 and 5 illustrate how changes over time can be examined, and how doses can be combined over a wide variety of sites. Figure 6- 8 were based on three years of doses analyzed together to take into account the periodic increase in occupational dose experienced during the refueling process at nuclear power plants. Figure 6 can be compared to Figures 7and 8 for examining differences due to radiation sources and job tasks. Figure 9 demonstrates how the Weibull approach can be combined with another performance indicator, namely collective dose to enhance the information provided by traditional and ALARA performance indicators.

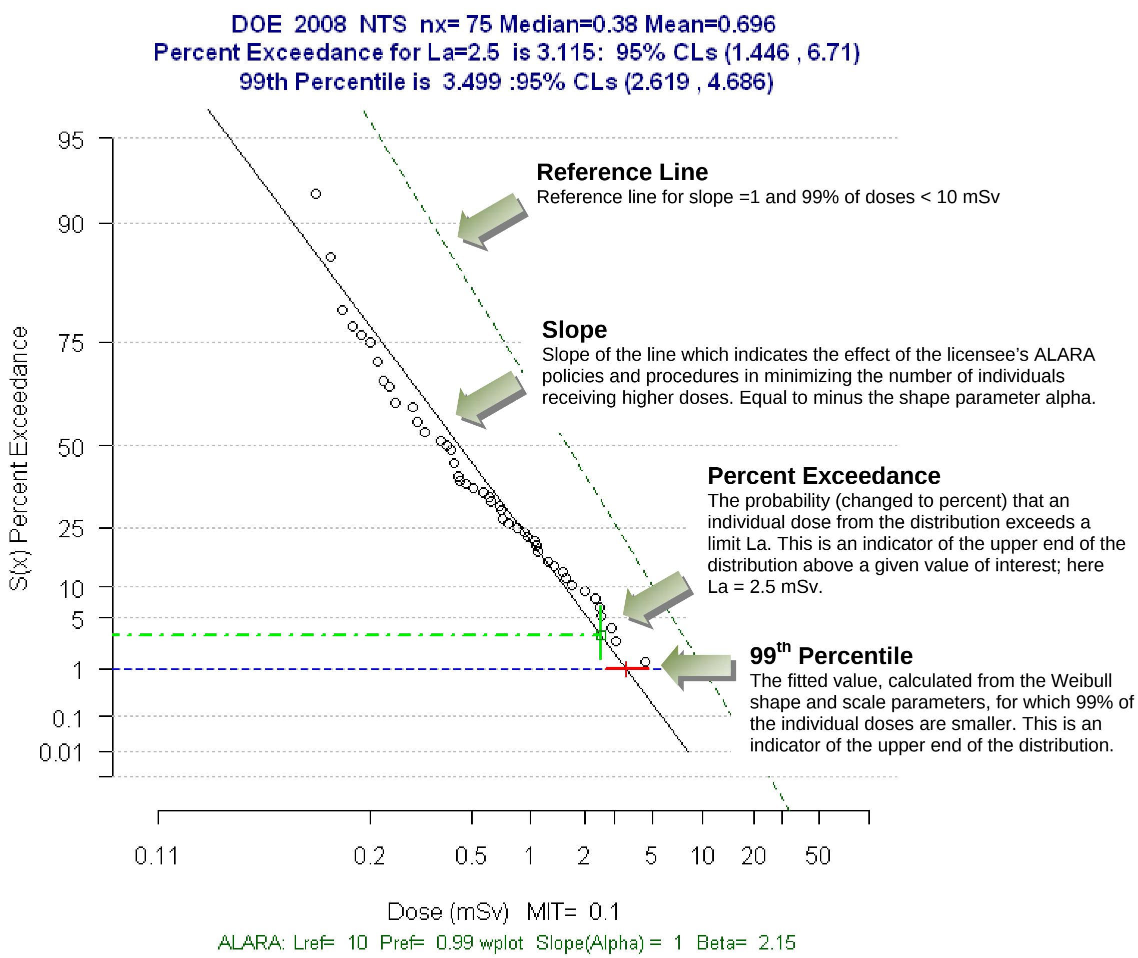
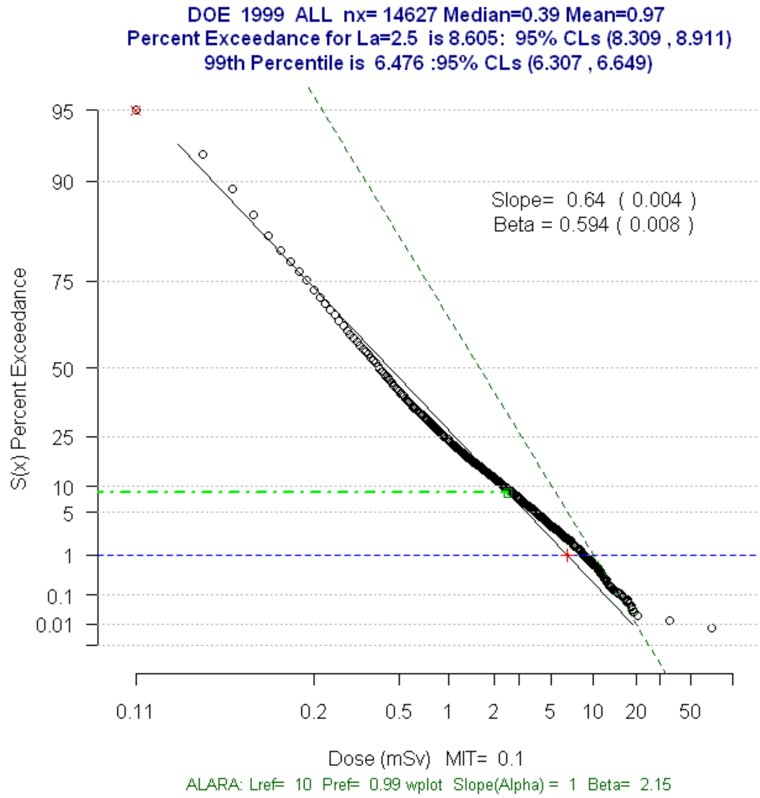


Figure 3: DOE NTS Site 2008



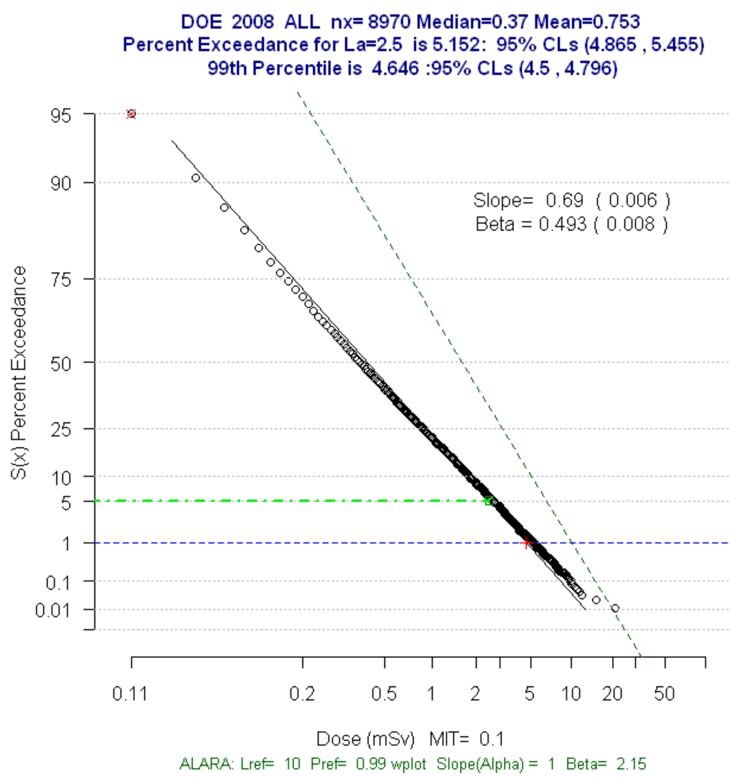
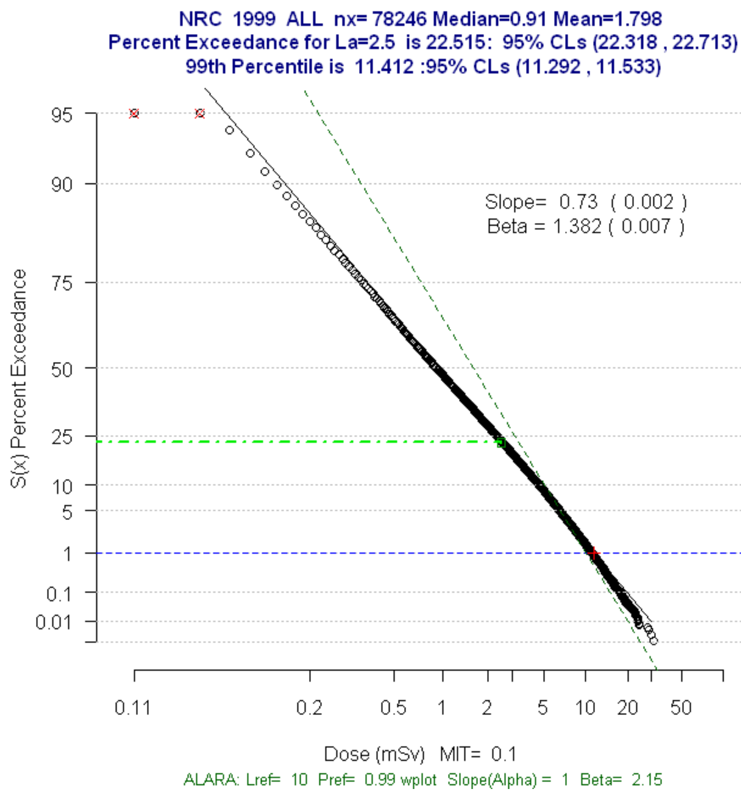


Figure 4: All DOE Sites Combined – 1999 and 2008



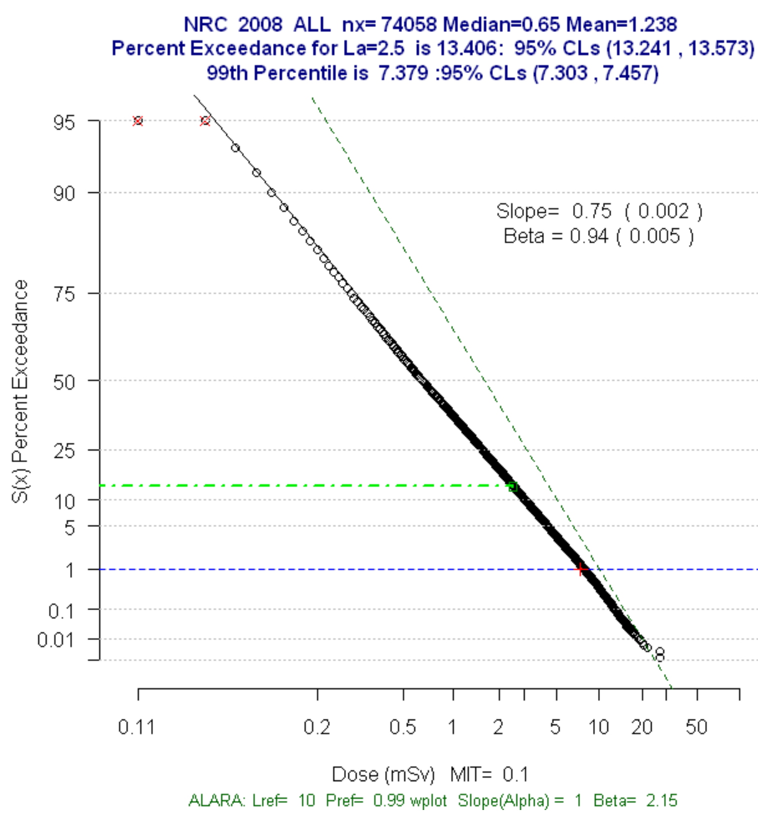


Figure 5: All NRC sites combined – 1999 and 2008

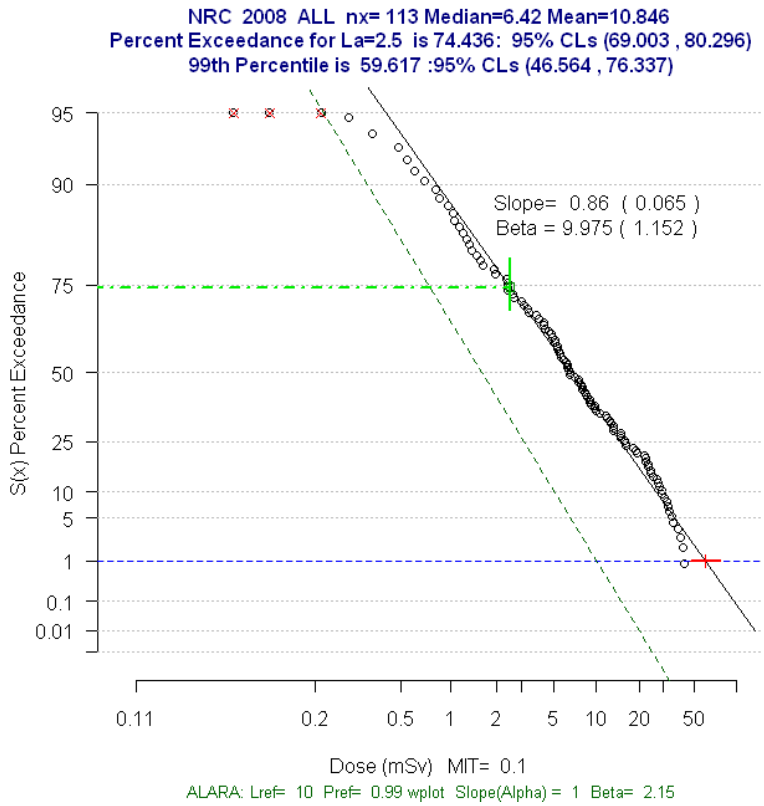


Figure 6: NRC Multi-Site Radiography Licensees

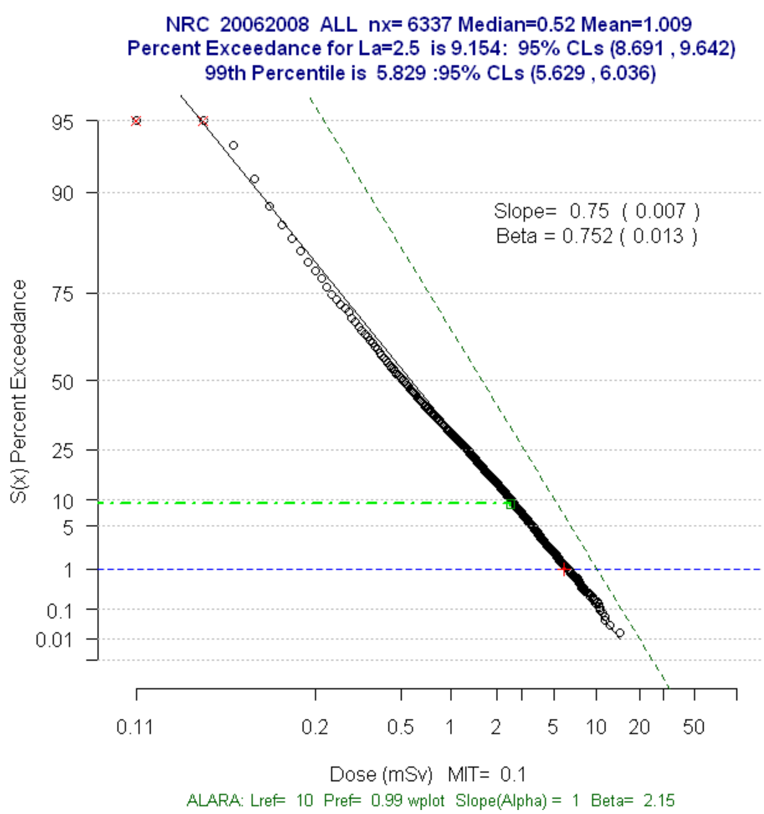


Figure 7: Susquehanna Nuclear Power Plant 2006-2008

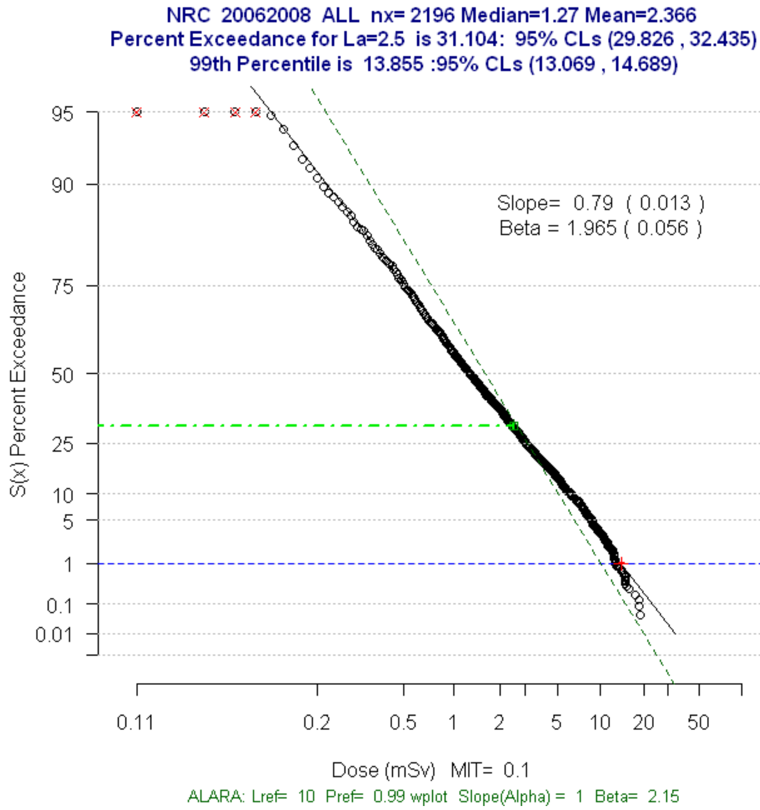


Figure 8: Palisades Nuclear Power Plant 2006-2008

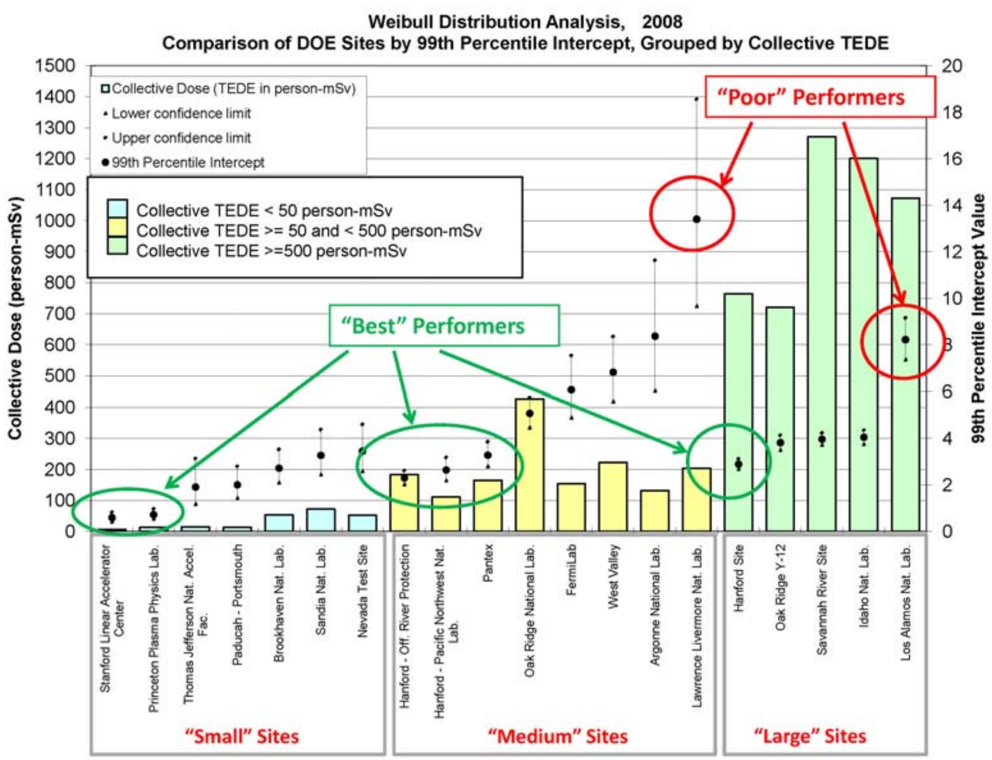


Figure 9: Using Collective Dose and Weibull Performance Indicators to evaluate ALARA among Sites

4. DISCUSSION

For the application of site radiation dose distributions discussed in this report, the Weibull model appears to fit the distributions effectively when the shape parameter is less than one. The distribution analysis approach using the Weibull distribution resulted in objective performance indicators, based on individual worker's doses, which can supplement the traditional indicators that are based on collective site information. Just as the traditional indicators collective dose and average measurable dose are both needed for appropriate interpretation, two distribution analysis indicators are recommended to obtain a balanced picture of the application of the ALARA principal. A slope greater than one, as opposed to less than one, indicates lack of ALARA effectiveness, and, in general, lower slopes are better. However, too shallow of a slope might be associated with a high 99^{th} percentile in rare instances of ALARA being applied effectively to most, but not all, workers. Therefore, a combination of slope and either the 99^{th} percentile or the exceedance fraction should be used together to evaluate ALARA. All of the results in this paper were obtained using the R (2008) environment for statistical computing. Software is currently being developed in R to facilitate the easy and efficient generation of the resultant performance indicators and plots.

5. CONCLUSIONS

The primary objectives of this research effort have been acomplished. The use of the Weibull distribution method has been shown to be applicable to the analysis of occupational radiation exposure and to fit the observed data when a minimum inclusion threshold is applied. The performance indicators based on the Weibull distribution (e.g., the slope, 99^{th} percentile, and exceedance fraction) provide enhanced, objective information for evaluating ALARA performance and can be used in conjunction with traditional indicators to provide a more in-depth view of radiation practices in an occupational exposure environment. Graphs have been developed to visually examine the statistical model and are useful for the comparison of ALARA practices between facilities or the trending of ALARA practices over time. With this approach, it is possible to analyze dose distributions from a wide variety of facilities engaged in different activities and varying in the number of exposed workers and magnitude of the collective dose. The broad application of this methodology can be seen as a significant addition to the traditional performance indicators that are dependent upon the magnitude of the collective dose and the size of the monitored workforce.

6. REFERENCES

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7. APPENDIX A: SELECTING A MINIMUM INCLUSION THRESHOLD

In selecting the MIT for application to occupational radiation exposure data, a key criterion was to choose the value that would retain as many of the doses as possible, since doses of zero or less after subtracting MIT are excluded from analysis.

The value of 0.1 mSv was selected for the MIT for the following reasons:

- Most DOE and NRC sites are currently accredited to a minimum detectable activity (MDA) of 0.1 mSv.
- Workers with TEDEs of 0.1 mSv or smaller are unquestionably not part of the scope of ALARA.
- The procedure of subtracting 0.1 mSv is conservative under ALARA since the lowest doses are not included in the dose distribution.
- Empirical evidence from recent DOE and NRC data demonstrates that doses of 0.1 mSv and below are not generally consistent with doses above this value. Selecting an MIT of 0.1 mSv appears to excludeTEDEs recorded for non-exposed workers as a result of background radiation and measurement error.

8. APPENDIX B: DETAILS OF WEIBULL ANALYSIS APPROACH

8.1. Obtaining Weibull Parameters by Maximum Likelihood

Let x_i , i = 1, …, n denote a random sample from the Weibull distribution with probability density function w(x; α, β) as defined in equation (2). The log of the likelihood function is

$$L(\alpha, \beta) = \sum_{i=1}^{n} \log \left[w(x_{i,} \alpha, \beta) \right] .$$

Maximum likelihood (ML) estimates – see Cohen (1965, 1991) and Cox and Hinkley (1979) -- of α and β are obtained using general purpose optimization procedure as implemented in the R (2008) function **optim** ( ) . A local R function is used to obtain result as follows:

1) Calculate initial estimates of the parameters using the method described by Menon (1963).
2) Use **optim** ( ) with the starting values from step 1, method = "Nelder-Mead". This is a robust optimization method that does require derivatives.
3) Use **optim** ( ) with ithe Nelder-Mead estimates from step 2, method = "L-BFGS-B" and Hessian = TRUE. This is a quasi-Newton method that uses a finite difference method to compute the gradient and allows box constraints on the parameters, i.e., Hessian = TRUE returns the Hessian matrix of second partial derivatives which is used to calculate an estimate of the variance-covariance matrix.

8.2. Creating a Weibull Probability Plot

A Weibull probability plot is obtained by linearizing the exceedance function, i.e. the exceedance function is transformed into the equation of a straight line of the form y = mx + b as follows:

$$S(u) = \exp(-[u/\beta]^{\alpha})$$
$$\ln(S(u)) = -[u/\beta]^{\alpha}$$
$$\ln(-\ln(S(u)) = -\alpha \ln(\beta) + \alpha \ln(u).$$

The S(u) has been transformed into the equation of a straight line with $y = \ln(-\ln(S(u))$ and $x = \ln(u)$, the y-intercept is $-\alpha \ln(\beta)$, and the slope of the line is the shape parameter α of the Weibull distribution. To emphasize the upper end of the dose distributions, the negative of the y-values were used in the ordered pairs for the graph, allowing the lines to be oriented from upper left to lower right, i.e., values of S(u) rather than F(u) are displayed on the vertical axis.

The following steps were used to obtain the Weibull probability plots:

- Calculate u_i, the distinct values of x in increasing order.
- Calculate $F(u_i)$, the empirical cumulative distribution function at u_i.
- Calculate $y_i = -\ln[-\ln(S_i)]$, where $S_i = [n/(n+1)][1-F_i]$ is the Weibull plotting position.
- Plot y_i versus $\ln(u_i)$.
- For $S_i = (0.01, 0.1, 1, 5, 10, 25, 50, 75, 90, 95)/100$ label the vertical axis with $100S_i$ at $-\ln[-\ln S_i]$.
- Label the horizontal axis with u_i at values of $\ln(u_i)$.

8.3. Calculating the 99th Percentile

For the 99th percentile the survival function S(x) = 1 – 0.99 = 0.01, i.e. 1% of the doses are higher than the 99th percentile. Because the y-axis of the Weibull plots is scaled in terms of the survival function, a horizontal line from 1% along the y-axis will cross the Weibull fit line at the point where the horizontal-axis value is the 99th percentile. In plotting units the value of $y = -\ln[-\ln(0.01)] = -1.527$.

$$S(u) = \exp(-(u/\beta)^{\alpha})$$
$$\ln(-\ln(0.01)) = \alpha \ln x_{0.99} - \alpha \ln \beta$$
$$\ln x_{0.99} = \ln \beta + (1.527/\alpha)$$
$$\ln x_{0.99} = \eta + 1.527\ \delta, \text{ where } \eta = \ln \beta \text{ and } \delta = (1/\alpha)$$
$$x_{0.99} = \exp(\eta + 1.527\ \delta)$$

The percentiles were calculated by using the relationship between the Weibull and the Gumbel Extreme Value.

8.4. Relationship between Weibull and Gumbel Extreme Value Distribution

If X follows the Weibull distribution with shape parameter α and scale parameter β as defined in equation (1), then it is well known (Lawless, 2003) that Y=lnX follows Gumbel distribution with probability density function

$$f(y) = (1/\delta)\exp[((y-\eta)/\delta) - \exp((y-\eta)/\delta)], \qquad (3)$$

where $\eta = \ln \beta$ and $\delta = (1/\alpha)$. The $100p^{th}$ percentile of (3) is $\ln(x_p) = y_p = \eta + k\delta$, where $k = \ln[-\ln(1-p)]$. Large sample confidence limits based on ML estimates of η and δ are equivalent to confidence limits based on estimates of β and α. Alternative methods for obtaining estimation and test procedures based on certain pivatol statistics have been described by Lawless (1974, 2003). For the special case of the 95th percentile, $S(x) = 0.05$ and $\ln(-\ln(0.05)) = 1.097 \simeq 1$. Therefore, $\ln x_{0.95} \simeq \varepsilon + \beta = \ln \beta + (1/\alpha)$.

8.5. Creating a Reference Line

To assist a site in tracking its ALARA effectiveness over time, a reference line may be useful. For example a reference line could represent an ALARA goal of 99% of the doses being less than a reference value Lr. After selecting a value for Lr, e.g. 10 mSv, a line can be determined by assuming a scale parameter equal to one (a negative exponential distribution) and solving for the Weibull scale parameter β. When $p = 0.99$ and $Lr = 10$, $\beta = [-10 / \ln(1-0.99)] = 2.17$.

8.6. Determining an Exceedance Fraction

An alternative parameterization of the survival function, convenient for calculating the exceedance fraction, is, for a specific limit L,

$$P[\,U > u] = S(u) = \exp(-\theta[u]^{\alpha}) = \exp(-\theta[x/L]^{\alpha}),\ \alpha > 0,\ \theta > 0 \qquad (4)$$

In this parameterization $\theta = (1/\beta)^{\alpha}$ and $u = x/L$. When $x = L$, $x/L = 1$, so $\Pr[X > L] = S(x{=}L) = \exp(-\theta)$. The probability that X exceeds a specified limit L is called the exceedance fraction for the limit L. To calculate an estimate of the exceedance fraction for a desired value of L, divide all doses by L before calculating the estimates of θ and its standard deviation. Then the exceedance fraction is $\exp(-\theta)$ and the standard deviation of θ can be used to determine the large sample confidence interval.